1 Formula Sheet

Newtons Law of Cooling/Heating:

$$\frac{dT}{dt} = k(T - T_1)$$

Useful Trig Identities:

$$\begin{aligned}
\sin^2(x) + \cos^2(x) &= 1 \\
\tan^2(x) + 1 &= \sec^2(x) \\
\sin^2(x) &= \frac{1 - \cos(2x)}{2} \\
\cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin(\cos^{-1}(x)) &= \sqrt{1 - x^2} \\
\cos(\sin^{-1}(x)) &= \sqrt{1 - x^2} \\
\sec(\tan^{-1}(x)) &= \sqrt{1 + x^2} \\
\tan(\sec^{-1}(x)) &= \begin{cases} \sqrt{x^2 - 1} & x \geq 1 \\ -\sqrt{x^2 - 1} & x \leq -1 \end{cases} \\
\sin(mx)\cos(nx) &= \frac{1}{2}[\sin(m+n)x + \sin(m-n)x] \\
\sin(mx)\sin(nx) &= -\frac{1}{2}[\cos(m+n)x - \cos(m-n)x] \\
\cos(mx)\cos(nx) &= \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]
\end{aligned}$$

Useful Derivatives:

$$\begin{aligned}
D_x \sin^{-1}(x) &= \frac{1}{\sqrt{1 - x^2}} \qquad -1 < x < 1 \\
D_x \cos^{-1}(x) &= -\frac{1}{\sqrt{1 - x^2}} \qquad -1 < x < 1 \\
D_x \tan^{-1}(x) &= \frac{1}{1 + x^2} \\
D_x \sec^{-1}(x) &= \frac{1}{|x|\sqrt{x^2 - 1}} \qquad |x| > 1
\end{aligned}$$

Some Useful Integral Forms:

$$\int \tan u du = -\ln|\cos(u)| + C$$

$$\int \cot(u) du = \ln|\sin(u)| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$$

Note: The practice midterm may be a little longer than the actual midterm, but it reflects problems similar in difficulty to what you may encounter in the actual exam.

2 True/False

1. The Power Rule of integration works for any integral of the form x^a where a is any real number.

2. We can find the slope of an inverse function even if we can't write down a formula for the inverse function.

3. $\sin(\sin^{-1}(3\pi)) = 3\pi$

4. As the step size h gets smaller (i.e we take more steps) Euler's method gets more accurate.

5. We can solve analytically (write down a formula/answer for) any integral of the form $\int \tan^m(x)\sec^n(x)dx$ where m,n are both any real numbers.

6. $y = -4x^5 + 3x - 4$ is an invertible function.

3 Free Response

Evaluate the Following Derivatives:

1. $\frac{d}{dx}[x^2 \ln(x) + e^{2x^2+3} - 3^x]$

2. $D_x[\sin(\sinh(3x))]$

3. find $\frac{dy}{dx}$ if $y = (x+1)^{\int_1^x 4x+3}$

4. $D_x[\sin^{-1}(3x^2 - 4x + \ln(x))]$

Evaluate the Following Integrals:

5. $\int \frac{x+1}{\sqrt{5-4x^2}} dx$

6. $\int \sin(x) \sinh(\cos(x)) dx$

7. $\int 4x^2 e^x dx$

8. $\int \tan^3(x)dx$

9. Salt water at a concentration of 2 pounds per gallon flows into a 100 gallon tank initially filled with fresh water (no salt) at 3 gallons per minute. If the fluid in the tank is constantly mixed and water flows out of the tank at the same rate that it flows in solve for how many pounds of salt are in the tank of water at t=10 minutes.